

Shear viscosity to entropy density ratio in the Boltzmann-Uehling-Uhlenbeck model

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The ratio of shear viscosity (η) to entropy density (s) for an equilibrated system is investigated in intermediate energy heavy ion collisions below 100A MeV within the framework of the Boltzmann-Uehling-Uhlenbeck (BUU) model. After the collision system almost reaches a local equilibration, the temperature, pressure and energy density are obtained from the phase space information and η/s is calculated using the Green-Kubo formulas. The results show that η/s decreases with incident energy and tend towards a smaller value around 0.5, which is not so drastically different from the BNL Relativistic Heavy Ion Collider results in the present model.

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I. INTRODUCTION

Studying the behavior of nuclear matter under extreme conditions is one of the most important problems in heavy ion collisions. Due to van der Waals nature of the nucleon-nucleon interaction, it is expected that multifragmentation may exhibit features of liquid-gas phase transition (LGPT) in intermediate energy heavy-ion collisions [1, 2]. Evidences of this have been provided from various observables, such as the nuclear caloric curve, fluctuation, fragment mass distribution and moment analysis etc [3–10]. Recent progress on nuclear liquid-gas phase transition has been reviewed, especially for the signals of LGPT in theory and experiment [11, 12].

Empirical observation of the temperature or incident energy dependence of the shear viscosity to entropy density ratio (η/s) for H₂O, He and Ne₂ exhibits a minimum in the vicinity of the critical point for phase transition [13]. Furthermore, a lower bound of $\eta/s > 1/4\pi$ obtained by Kovtun-Son-Starinets (KSS) in certain gauge theories is speculated to be valid for several substances in nature [14, 15]. In ultra-relativistic heavy ion collision [16–21], people used the shear viscosity to entropy density ratio to study the Quark-Gluon Plasma phase and get the minimum value of η/s , so it is very interesting to study shear viscosity or η/s in intermediate energy heavy ion collision [22–26]. Unlike the studies on η/s at relativistic energies, there is still very limited investigations in intermediate energy heavy-ion collisions.

In this work, we study the thermodynamic and transport properties of nuclear reaction and try to see how the η/s evolves with the beam energy or temperature in a transport model. We study the equilibration of nuclear system within a finite volume using BUU model. In order to make the system contain enough number of nucleons in the fixed spherical volume, we choose Au + Au system in head-on collision ($b = 0$ fm). The system evolves with

time for long enough time so that it is in freeze-out stage.

In the final stage of the central collisions, the system can be viewed as locally equilibrated. The equilibrium in intermediate energy heavy ion collisions can be judged by using the temperature and other dynamical variables [27]. After the system is in equilibrium, we calculate the thermodynamic parameters (pressure, energy density and entropy density) from phase space information of the system. Shear viscosity coefficient is calculated from stress tensor fluctuations around the equilibrium state using Green-Kubo formula [25, 28]. Finally, we compare η/s in different incident energy with different nuclear equation of state and discuss the results.

The rest of the paper is organized as follows: In Sec. II, we describe the situation of system equilibrium. In Sec. III, we calculate viscosity coefficient and entropy density. Finally, a brief summary and outlook is made in Sec. IV.

II. EQUILIBRATION OF FINITE NUCLEON SYSTEM

We calculate the shear viscosity to entropy density ratio η/s of an equilibrated nuclear system in intermediate energy heavy ion collisions using BUU model, which is a one-body microscopic transport model based upon the Boltzmann equation [29, 30].

The BUU equation reads [31]

$$\begin{aligned} \frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla_r U \cdot \nabla_p f = & \frac{4}{(2\pi)^3} \int d^3 p_2 d^3 p_3 d\Omega \\ \frac{d\sigma_{NN}}{d\Omega} V_{12} \times [f_3 f_4 (1-f)(1-f_2) - f f_2 (1-f_3)(1-f_4)] \\ & \delta^3(p + p_2 - p_3 - p_4). \end{aligned} \quad (1)$$

It is solved with the method of Bertsch and Das Gupta [32]. In Eq.(1), $\frac{d\sigma_{NN}}{d\Omega}$ and V_{12} are in-medium nucleon-nucleon cross section and relative velocity for the colliding nucleons, respectively, and U is the mean field poten-

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tial including the isospin-dependent term:

$$U(\rho, \tau_z) = a\left(\frac{\rho}{\rho_0}\right) + b\left(\frac{\rho}{\rho_0}\right)^\sigma + C_{sym} \frac{(\rho_n - \rho_p)}{\rho_0} \tau_z, \quad (2)$$

where ρ_0 is the normal nuclear matter density; ρ , ρ_n , and ρ_p are the nucleon, neutron and proton densities, respectively; τ_z equals 1 or -1 for neutrons and protons, respectively; The coefficients a , b and σ are parameters for nuclear equation of state. Two sets of mean field parameters are used in this work, namely the soft EOS with the compressibility K of 200 MeV ($a = -356$ MeV, $b = 303$ MeV, $\sigma = 7/6$), and the hard EOS with K of 380 MeV ($a = -124$ MeV, $b = 70.5$ MeV, $\sigma = 2$). C_{sym} is the symmetry energy strength due to the density difference of neutrons and protons in nuclear medium, here $C_{sym} = 32$ MeV is used.

In this work, we focus on the thermodynamic and transport properties of a nuclear system. For this purpose, we investigate the process of the head-on Au + Au collision in a spherical volume with the radius of 5 fm. In the following calculation, we show the results with the hard EOS except in the case we mentioned.

First we check the evolution of equilibration situation and temperature. The anisotropy ratio, which is a measure of the degree of equilibration reached in a heavy-ion reaction, is defined as

$$R_p = \frac{2 R_{\parallel}}{\pi R_{\perp}}, \quad (3)$$

where $R_{\parallel} = \langle \sqrt{p_x^2 + p_y^2} \rangle$ and $R_{\perp} = \langle \sqrt{p_z^2} \rangle$ are calculated by the momentum of nucleons in the given sphere. As an example, the time evolutions of R_p for Au+Au systems within a 5fm-radius sphere at 50 A MeV are shown in Fig. 1(a). When R_p approaches to 1 at around 100 fm/c, nuclear system is under equilibrium.

Time evolution of temperature is also used to judge the state of equilibration. Temperature of the system can be derived from the momentum fluctuations of particles in the center of mass frame of the fragmenting source [33]. The variance σ^2 is obtained from the Q_z distribution through

$$\sigma^2 = \langle Q_z^2 \rangle - \langle Q_z \rangle^2, \quad (4)$$

where Q_z is the quadruple moment which is defined by $Q_z = 2p_z^2 - p_x^2 - p_y^2$, and p_x, p_y and p_z are three components of momentum vector extracted from the phase space of BUU model. If the mean equals zero, the second term vanishes. Q_z^2 is described by

$$\langle Q_z^2 \rangle = \int d^3p (2p_z^2 - p_x^2 - p_y^2)^2 f(p). \quad (5)$$

Assuming a Maxwellian distribution for the momentum distribution, i.e.

$$f(p) = \frac{1}{(2\pi mT)^{3/2}} e^{-\frac{p_x^2 + p_y^2 + p_z^2}{2mT}}, \quad (6)$$

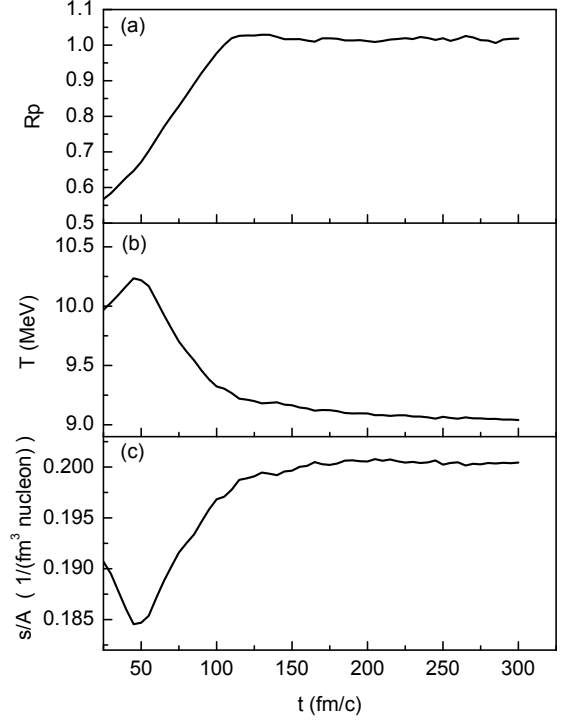


FIG. 1: R_p (a), temperature (b) and entropy density per nucleon (c) as a function of time (after 24 fm/c) for the head-on Au + Au collision within 5fm-radius sphere at 50 A MeV.

we can obtain

$$\langle Q_z^2 \rangle = 4m^2 A^2 T^2 \quad (7)$$

after Gaussian integral, where m is the mass of a nucleon and A is the mass number of the fragment. For a nucleonic system, we have $A = 1$ and can calculate the evolution of temperature using this equation. Fig. 1(b) shows the temperature's evolution after 25 fm/c, it is seen that temperature reaches a maximum around 50 fm/c when the system is in the most compressible stage and then it starts cool down when the system expands, later on the system tends thermodynamic equilibrium. For an equilibrated system, the kinetic energy distributions approach the Boltzmann distribution as time increases [34]. After the expansion process, the system will approach an equilibrate state, so we can investigate the viscosity coefficient and entropy density in system.

Except temperature, other thermodynamic variables can be calculated during heavy-ion collisions. Energy density inside a volume with the 5fm radius can be defined as

$$\varepsilon = \frac{1}{V} \sum_{r_i < r_0} E_i, \quad (8)$$

where E_i is $\sqrt{p_i^2 + m_i^2}$, r_i is the position of the i -th nucleon in the center of mass and r_0 is the selected radius (here we set $r_0 = 5$ fm) and pressure can be defined as

$$P = \frac{1}{3V} \sum_{r_i < r_0} \frac{p_i^2}{E_i}. \quad (9)$$

After we get the energy density, pressure and temperature, entropy density can be calculated by the Gibbs formula

$$s = \frac{\varepsilon + P - \mu_n \rho}{T}, \quad (10)$$

where μ_n is the nucleon chemical potential and ρ is nucleon density of system within the given sphere. In principal, once we have the temperature T and $f(p)$, we can fit to a Fermi-Dirac function to extract the chemical potential. However, we can assume, for simplicity, zero nucleon chemical potential, or μ_n can be taken around 20 MeV in the present calculation [35]. In the following calculations of entropy and η/s , we only show the results with $\mu_n = 20$ MeV. But we have also checked the results with zero chemical potential, this will increase the entropy about 8% and then lead to the decreasing of η/s about 8%. However, this does not change our conclusions of this work. Fig. 1(c) shows the entropy density per nucleon (s/A) evolves with time after 25 fm/c. It seems the entropy density per nucleon reaches a minimum when the system is in the most compression stage and rises up in the expansion phase, it finally reaches to an asymptotic value. From the viewpoint of phase space, the number of occupied states is most limited in the high density phase. In the high density phase, Fermi blocking forces some of the nucleons into higher momentum states but still in a spatially confined region, which makes the entropy density the minimum. When the system expands, the Fermi blocking is reduced, the system cools, but more coordinate volume is occupied. Therefore it is possible that this leads to a non-isentropic expansion until the system is in the freeze-out stage.

Energy density and temperature can be calculated in different time when the reaction is going on. Fig. 2(a) shows energy density per nucleon versus temperature for the studied system after 25 fm/c. Similarly, pressure per nucleon increases with temperature is shown in Fig.2(b). From the figure, we can see both energy density and pressure increase with temperature. This can be understood that in a given volume, the increasing of temperature reflects stronger thermal motion of nucleons, therefore the kinetic energy will give more contribution on energy density and pressure.

III. VISCOSITY COEFFICIENT AND ENTROPY DENSITY

Now let's move on the investigation on transport coefficient of Au+Au system in a given volume by the

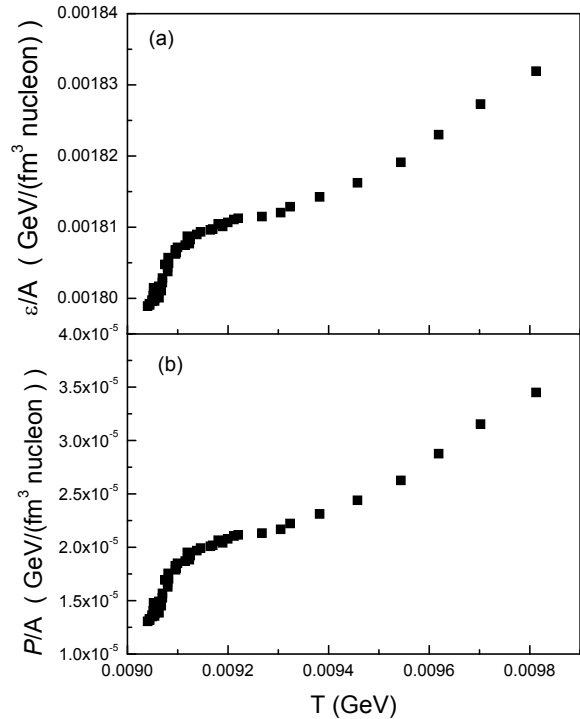


FIG. 2: Energy density per nucleon (a) and pressure per nucleon (b) as a function of temperature for the head-on Au + Au collision within 5fm-radius sphere at 50 A MeV.

BUU model. Viscosity is one of the transport coefficients which characterize the dynamical fluctuation of dissipative fluxes in a medium. Transport coefficients can be measured, as in the case of condensed matter applications. Also, they should be in principle calculable from the first principle. Monte-Carlo simulation for transport coefficients is a powerful tool when studying transport coefficients using Green-Kubo relations [36, 37]. In high energy heavy-ion collisions, calculation of transport coefficients of shear viscosity for a binary mixture [38], and the calculation of coefficient of a hadrons gas has been studied [34, 39]. The situation of nuclear gas in intermediate energy heavy ion collisions is similar to hadrons gas. To study the extended irreversible dynamic processes, we use the Kubo fluctuation theory to extract transport coefficients [28]. The formula relates linear transport coefficients to near-equilibrium correlations of dissipative fluxes and treats dissipative fluxes as perturbations to local thermal equilibrium. The Green-Kubo formula for shear viscosity is defined by

$$\eta = \frac{1}{T} \int d^3r \int_0^\infty dt \langle \pi_{ij}(0,0) \pi_{ij}(\vec{r}, t) \rangle, \quad (11)$$

where T is the equilibrium temperature of the system, t is the post-equilibration time (the above formula defines

$t = 0$ as the time the system equilibrates and is determined by equilibrium time), and $\langle \pi_{ij}(0,0)\pi_{ij}(\vec{r},t) \rangle$ is the shear component of the energy momentum tensor. The expression for the energy momentum tensor is defined by $\pi_{ij} = T_{ij} - \frac{1}{3}\delta_{ij}T_i^i$, the momentum tensor reads [34]

$$T_{ij}(r,t) = \int d^3p \frac{p^i p^j}{p^0} f(x,p,t), \quad (12)$$

where $f(x,p,t)$ is the phase space density of the particles in the system. In order to compute an integral, we assume that nucleons are uniformly distributed in the space. Meanwhile, the isolated spherical volume with the radius of 5 fm is fixed, so the viscosity becomes

$$\eta = \frac{V}{T} \langle \pi_{ij}(0)^2 \rangle \tau_\pi, \quad (13)$$

where τ_π is calculated by

$$\langle \pi_{ij}(0)\pi_{ij}(t) \rangle \propto \exp\left(-\frac{t}{\tau_\pi}\right). \quad (14)$$

As shown in Fig. 3(a), $\langle \pi_{ij}(0,0)\pi_{ij}(\vec{r},t) \rangle$ is plotted as a function of time for Au + Au at 50 A MeV. The correlation function is damped exponentially with time and can be fitted by the Eq. (14) to extract the inverse slope corresponds as the relaxation time. Fig. 3(b) summarizes the relaxation time decrease as the increase of incident energy, indicating that the system can approach to equilibrium faster at higher incident energy.

Using the above method, we present the value of η/s as a function of incident energy after the studied system has been in equilibrium in Fig. 4. The two sets of nuclear equation of state are used. The η/s value shows a rapid fall as the increasing of incident energy up to $E < 70$ A MeV and then drops slowly to a value close 0.5 when $E > 70$ A MeV. Since the BUU equation is one-body theory, fragmentation which originates from the fluctuation and correlation can not be treated in the present model. In this case, the phase transition behavior cannot be predicted in the BUU model. The continuous drop of the ratio of shear viscosity to entropy density does not show a minimum at a certain beam energy, which indicates no obvious phase change or critical behavior in the present model. This is a shortcoming of the BUU model itself, especially when it is applied to higher beam energy. Actually, when we calculate the differential values of η/s versus the beam energy (see the inset of Fig. 4), it seems a turning point around $E \sim 65$ A MeV. This turning point could indicate the change of dynamical behavior of system, in other words, other mechanisms may be needed to be taken into account in the model especially at higher beam energies, eg. multifragmentation [40]. Alternatively, lack of minimum η/s perhaps shows that a behavior with a local minimum of η/s at phase transition temperature might not be universal [41]. In the present BUU calculation, all calculated values of η/s are well above the conjectured KSS lower bound of $1/4\pi$ [14, 15]. Comparing these values of η/s for our finite

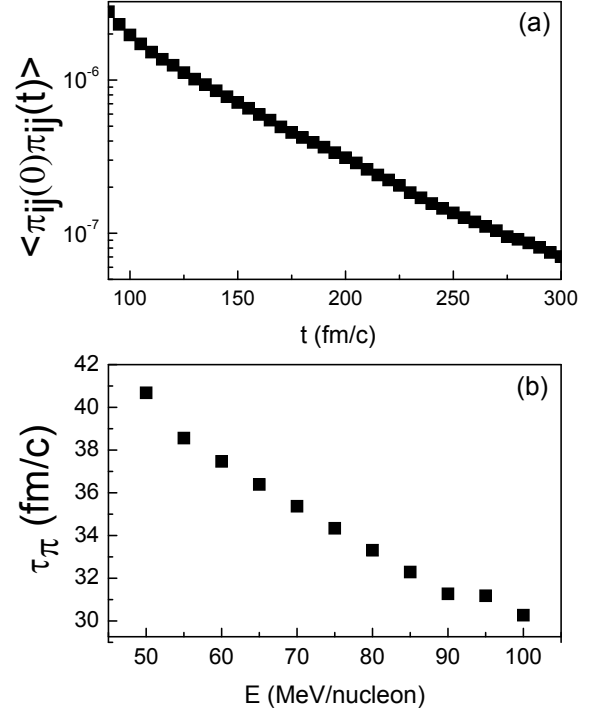


FIG. 3: (a) $\langle \pi_{ij}(0,0)\pi_{ij}(\vec{r},t) \rangle$ evolves with time for the head-on Au+Au collision in a given 5fm-radius volume at 50 A MeV; (b) Relaxation time as a function of incident energy for the head-on Au+Au collision in a given 5fm-radius volume.

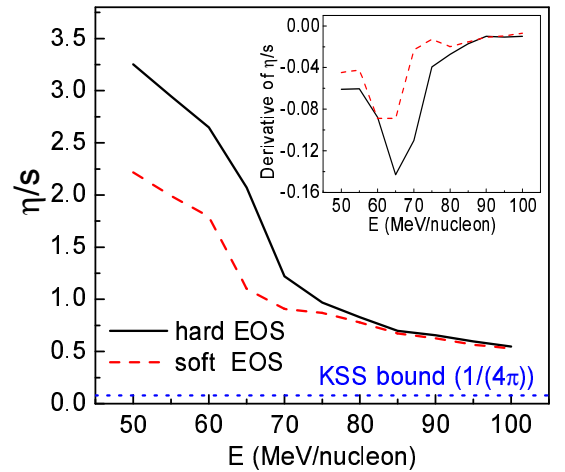


FIG. 4: (Color online) η/s as a function of beam energy for the head-on Au + Au collision in a spherical volume with radius of 5 fm. The inset shows the derivative of η/s versus beam energy.

nuclei in BUU model, we see they are not drastically different either from the RHIC results [17, 42] or from the results of the usual finite nuclei at low temperature from the widths of giant vibrational states in nuclei [26]. As pointed in Ref. [26], it is possible that the strong fluidity is a characteristic feature of the strong interaction of the many-body nuclear systems in general and not just of the state created in the relativistic collisions. Another interesting point from Fig. 4 is that η/s shows EOS sensitivity in lower beam energy: hard EOS displays larger η/s than the soft one, i.e. larger compressibility of nuclear matter can lead to higher η/s value.

IV. SUMMARY

In summary, we studied thermodynamic variables as well as viscosity and entropy density for heavy ion collision after the system tends towards equilibrium in intermediate energy heavy ion collisions in the framework of BUU model. The Green-Kubo relation has been applied for the nucleonic matter in a central region with a moderate volume when the system has been in equilibrium stage for central heavy-ion collisions of Au + Au. It is found that the ratio of shear viscosity to entropy density η/s decreases very quickly before 70 A MeV and then drops slowly towards a smaller value of η/s around 0.5 at higher beam energy in Au+Au system. The η/s are not drastically different either from the RHIC results or from the results of the usual finite nuclei at low temperature. However, no obvious minimum η/s value oc-

curs within the investigated energy range. This may reflect that no liquid-gas phase transition behavior is displayed in the present model due to the shortcoming of the model itself which lacks dynamical fluctuation and correlation. Relating the shortcoming, the equilibrium temperature could be a little higher than other model (QMD, SMM) which considers fragment formation [43] and while the shear viscosity and entropy density might be influenced by the cluster formation. Therefore, other models which can incorporate liquid gas phase transition should be checked for the shear viscosity and entropy density. For instance, it will be very interesting to use quantum molecular dynamics-type model to check if a minimum of η/s will occur around liquid-gas phase transition. The work along this direction is in progress. Of course, experimental studies on shear viscosity is more important to demonstrate the relation of η/s and liquid-gas transition point.

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